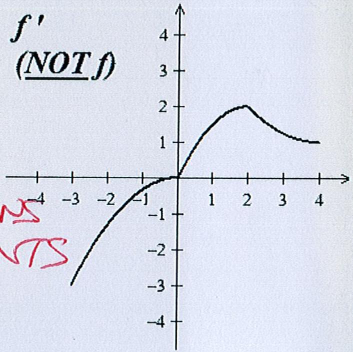
$f(x)$ is a continuous function whose derivative $f'(x)$ is shown on the right.

SCORE: _____ / 4 PTS

The following questions are about the function f, **<u>NOT THE FUNCTION f'</u>**.



[a] Write "I UNDERSTAND" if you understand that the following questions are about the continuous function f, **<u>NOT THE FUNCTION f'</u>**.

☆ YOU MUST GIVE THE REASONS IN ORDER TO GET ANY POINTS

[b] Find all intervals over which f is decreasing.
<u>Justify your answer very briefly.</u>

☆ ① $f' < 0$ ON $(-3, 0)$

[c] Find all intervals over which f is concave up.
<u>Justify your answer very briefly.</u>

☆ ① f' INCR ON $(-3, 2)$

[d] Find the $x-$ coordinates of all local extrema of f and identify whether they are local maxima or minima.
<u>Justify your answer very briefly.</u>

① $f' = 0$ AND CHANGES FROM $-$ TO $+$ @

① $x = 0$ LOCAL MIN

$f(x)$ is a polynomial function with critical numbers -3 and -1, and second derivative **SCORE: _____ / 3 PTS**

$f''(x) = (5x+7)(x+3)\text{^}3$. Run the Second Derivative Test for each critical number, and state what it tells you about that critical number.

Justify your answer very briefly. Do NOT use the First Derivative Test.

$x = -3$: $f'' = 0$ NO CONCLUSION 1

$x = -1$: $f'' > 0$ LOCAL MIN 2

$f(x)$ is a continuous function with derivative $f'(x) = (x+1)^{-\frac{1}{3}}(2x-5)^4$. **SCORE: _____ / 5 PTS**

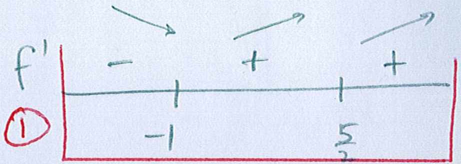
[a] Find the critical numbers of f. **<u>Justify your answer very briefly.</u>**

★ ① $f'(x)$ DNE @ $x = -1$,

★ ($\frac{1}{2}$) $f'(x) = 0$ @ $x = \frac{5}{2}$

★ YOU MUST GIVE THE REASONS IN ORDER TO GET ANY POINTS

[b] Run the First Derivative Test for each critical number, and state what it tells you about that critical number.
<u>Justify your answer very briefly. Do NOT use the Second Derivative Test.</u>



f' CHANGES FROM − TO + ★ ① @ $x = -1$, ① LOCAL MIN,

f' DOES NOT CHANGE SIGNS @ $x = \frac{5}{2}$ NEITHER LOCAL MAX NOR MIN ★ ($\frac{1}{2}$)

Graph $f(x) = 18x(4-x)^{\frac{1}{3}}$ using the process shown in lecture and in the website handout. **SCORE: _____ / 18 PTS**

Complete the table at the bottom of the page, after showing relevant work (you do NOT need to show work for entries marked ★).
You will NOT receive credit for the entries in the table if the relevant work is missing.

NOTE: $f''(x) = (8x-48)(4-x)^{-\frac{5}{3}}$

X-INT: $18x(4-x)^{\frac{1}{3}} = 0 \longrightarrow x = 0, 4$

y-INT: $f(0) = 0$

$\frac{1}{2}$ $\lim\limits_{x \to \infty} 18x(4-x)^{\frac{1}{3}} = -\infty$, $(\infty \cdot -\infty)$

$\frac{1}{2}$ $\lim\limits_{x \to -\infty} 18x(4-x)^{\frac{1}{3}} = -\infty$ $(-\infty \cdot \infty)$

$f'(x) = 18(4-x)^{\frac{1}{3}} + \overset{6}{\cancel{18}}x \cdot \frac{1}{\cancel{3}}(4-x)^{-\frac{2}{3}} \cdot (-1)$ 1

$= 6(4-x)^{-\frac{2}{3}}(3(4-x) - x)$

$= 6(4-x)^{-\frac{2}{3}}(12 - 4x)$

$= 24(3-x)(4-x)^{-\frac{2}{3}}$ 1

$\frac{1}{2}$ f' DNE @ $x = 4$, $f' = 0$ @ $x = 3$ $\frac{1}{2}$

$\frac{1}{2}$ f'' DNE @ $x = 4$, $f'' = 0$ @ $x = 6$ $\frac{1}{2}$

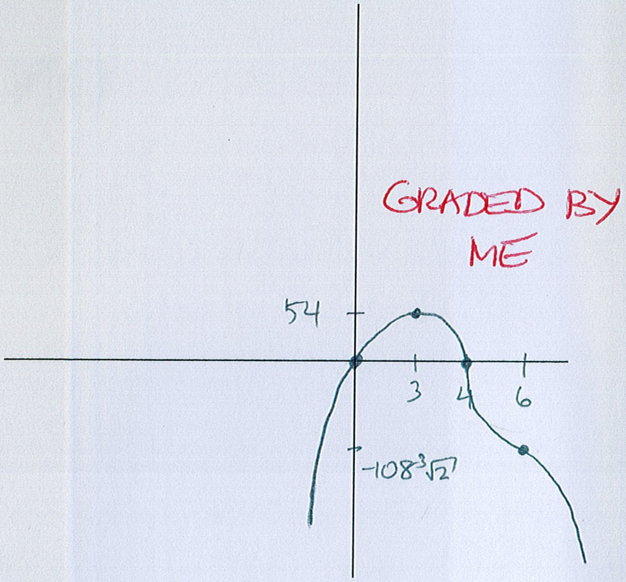
1 $\lim\limits_{x \to 4^-} f'(x) = \lim\limits_{x \to 4^-} \frac{24(3-x)}{(4-x)^{\frac{2}{3}}} = -\infty$ $\left(\frac{-24}{0^+}\right)$

1 $\lim\limits_{x \to 4^+} f'(x) = \lim\limits_{x \to 4^+} \frac{24(3-x)}{(4-x)^{\frac{2}{3}}} = -\infty$ $\left(\frac{-24}{0^+}\right)$

2

		3		4		6	
f'	+	HTL MAX	−	VTL	−		−
f''	−		−	IP	+		−
	╱		╲		╲		╲
		(3,54)		(4,0)		$(6, -108\sqrt[3]{2})$	

GRADED BY ME



★ Domain	★ Discontinuities	Intercepts (specify $x-$ or $y-$)	One sided limits at each discontinuity (write using proper limit notation)	
$\frac{1}{2}$ $(-\infty, \infty)$	NONE	X-INT: 0, 4 $\frac{1}{2}$ y-INT: 0	N/A	
Equations of Horizontal Asymptotes	Intervals of Increase	Intervals of Decrease	Intervals of Upward Concavity	Intervals of Downward Concavity
$\frac{1}{2}$ NONE	$(-\infty, 3)$ $\frac{1}{2}$	$(3, \infty)$ $\frac{1}{2}$	$(4, 6)$ $\frac{1}{2}$	$(-\infty, 4)$ $(6, \infty)$ 1
Vertical Tangent Lines (x-coordinates)	Horizontal Tangent Lines (x-coordinates)	Local Maxima (x-coordinates)	Local Minima (x-coordinates)	Inflection Points (x-coordinates)
4 $\frac{1}{2}$	3 $\frac{1}{2}$	3 $\frac{1}{2}$	NONE $\frac{1}{2}$	4, 6 1